Description of Colombian Electricity Pricing Derivatives

Dr. S Prabakaran,
Associate Professor,
School of Economics and Business Administration,
Department of Accounting & Finance,
Pontificia Universidad Javeriana Cali. Cali, Colombia.
Email – jopraba@gmail.com

Abstract

Electricity markets are becoming a popular field of research amongst academics because of the lack of appropriate models for describing electricity price behavior and pricing derivatives instruments. Models for price dynamics must consider seasonality and spiky behavior of jumps which seem hard to model by standard jump process. Without good models for electricity price dynamics, it is difficult to think about good models for futures, forward, swaps, and option pricing. In this paper, we attempt to introduce an algorithm for pricing derivatives to intuition from the Colombian electricity market. The main ambition of this study is fourfold: 1) First we begin our approach through to simple stochastic models for electricity pricing. 2) Next, we derive analytical formulas for the prices of electricity derivatives with different derivatives tools. 3) Then we extent short of the model for price risk in the electricity spot market 4) Finally we construct the model estimation under the physical measures for the Colombian electricity market. And this paper ends with a conclusion.

Key Words - Electricity markets, Energy Derivative, Option, and Forward Contract.

I. INTRODUCTION

Deregulation of electricity markets has led to a substantial increase in risk borne by market participants. The often unexpected, extreme spot price changes range even two orders of magnitude and can cause severe financial problems to the utilities that buy electricity in the wholesale market and deliver it to consumers at fixed prices. The utilities and other power market companies need to hedge against this price risk. A straightforward way to do it is to use derivatives, like forwards and options.

Here, we use the latter approach and describe the spot price dynamic models to review the electricity pricing with details of how to implement the pricing of the electricity market an After specifying a model we have to choose for derivatives pricing methodology. However, such an approach fails in the case of electricity due to extremely limited storage possibilities. Therefore, instead of using a discrete-time model approach we employ a concept of the risk premium/market price of risk and find such a pricing measure that yields the observed forward market prices. With such methodology, we can derive forward prices from the spot price model and to find explicit formulas for premiums of European options written on spot, as well as, on forwarding prices.

Over the last two decades, the electricity industry worldwide has undergone a profound restructuring process. Particularly in Colombia, laws 142 and 143 of 1994 in addition to later reforms, opened up an intensive reorganization of the electricity market, like the creation of the wholesale electricity market (MEM) in 1995 simultaneously to the vertical unbundling of the generation, transmission, distribution, and retail activities, seeking to improve the efficiency and quality of the electricity industry. Under this new framework, the power generation and retail businesses could be competitive deregulated markets, whereas the remaining two, transmission and distribution, were established as regulated activities.

Colombia has a hydro-dominated electricity market. Roughly 80% of its energy comes from hydro resources, 67% of its capacity, and 50% of its firm energy—energy in an exceptionally dry period. The cornerstones of the wholesale electricity market in Colombia are the spot energy market and the firm energy market. The spot energy market is a single-zone hourly market that determines the spot energy price in every hour as well as the efficient dispatch of resources.

This paper aims to introduce pricing electricity derivatives to the Colombian market with an alternative formalism.

II. STOCHASTIC MODEL FOR ELECTRICITY PRICING DERIVATIVES

In the last few years, there has been a rapidly increasing literature on stochastic models for the prices of electricity and other commodities. Many researchers have observed that the models typically used in financial markets are inappropriate due to the special features of commodity prices and especially of electricity prices as described in the introduction.

In this section, we will give a short evaluation of some of the models considered so far in the literature and compare them with our approach.

The choice of the stochastic model for electricity prices depends on the time granularity that needs to be reflected in the model. Liquidly traded futures and forward contracts typically have full months, quarters, or years as delivery periods, either as base load or peak load. Price quotes for single hour deliveries are in most cases only available as day-ahead prices from the spot market. However, many structured OTC products, such as swing options, are strongly influenced by the hourly price behavior. Since due to the non-storability of electricity, spot products cannot be used for hedging purposes, the electricity market is a highly incomplete market and pure arbitrage option pricing methods fail for most structured products. Previous work has been focused mainly on either of the two following approaches:

- Market models for futures prices: Instead of modeling the spot price and deriving futures prices, the futures prices themselves are modeled. This approach goes back to Black's model [1], where a single futures contract is considered. Ideas from the

IEECP'21, July 29-30, 2021, Silicon Valley, San Francisco, CA – USA

DOI : https://dx.doi.org/10.6084/m9.figshare.14735834

Heath-Jarrow-Morton theory for interest rates [2] are used in [3], [4], [5] and [6] to model the dynamics of the whole futures price curve. Such models have the advantage that the market can be considered as being complete and standard risk-neutral pricing may be used. Risk-neutral parameters can often be implied from traded options on futures prices. The disadvantage of such approaches is that futures prices do not reveal information about price behavior on an hourly or even daily time scale.

- Spot price models: This class of models aims at capturing the hourly price behavior by fitting their model to historical spot price data. Since there is no arbitrage relation between spot prices and futures prices, additional assumptions have to be made to use this model for pricing derivatives. Usually, this is done either by assuming the rational expectation hypothesis

$$F_{t,T}=\mathrm{E}\left[S_T\,|\,F_t\right], \tag{1}$$

As done e.g. in [7] to price generation assets, or by calibrating a market price of risk for each factor and then changing to an equivalent martingale measure P^* under which the relation $F_{t,T}=\mathrm{E}^*\left[S_T\,|\,F_t\right]$, holds.

Most models for the spot market employ at least two risk factors: one factor capturing the short-term hourly price dynamics characterized by mean reversion and extremely high volatility, and the other factor representing long-term price behavior observed in the futures market. Since there are no liquidly traded derivatives on a daily or hourly time scale that have a strong dependence on the short-term risk factor, it is exceedingly difficult to estimate the short-term market price of risk.

Through this paper, we will denote S_t the spot market price at time *t*. Since are working in a deterministic interest rate framework, we will not distinguish between forward and futures prices. Therefore, single hour futures prices at time *t* for delivery at time *T* are conditional expectations under the equivalent martingale measure

$$F_{t,T}=\mathrm{E}^*\left[S_T\,|\,F_t\right], \tag{2}$$

Where $F_t=\sigma\left[S_s\text{:s} \le \text{t}\right]$, is the natural filtration generated by the price process?

Future prices for power delivery over a period $S_t=(\ \ )\left[T_1,T_2\right]$ are given by

$$F_{t,T_1,T_2}=\mathrm{E}^*\left[\frac{1}{T_2\text{-}T_1}\int_{T_1}^{T_2} S_T dT \middle| F_t\right]=\frac{1}{T_2\text{-}T_1}\int_{T_1}^{T_2} F_{t,T} dT, \tag{3}$$

Or, in a discrete time setting by

$$F_{t,T_1,T_2}=\mathrm{E}^*\left[\frac{1}{T_2\text{-}T_1}\sum_{T=T_1}^{T_2\text{-}1} S_t \middle| F_t\right]=\frac{1}{T_2\text{-}T_1}\sum_{T=T_1}^{T_2\text{-}1} F_{t,T}\,. \tag{4}$$

The simplest model considering mean-reverting behavior is given by an Ornstein- Uhlenbeck process. Here the price process S_t is a diffusion process satisfying the stochastic differential equation

$$dS_t=\ -\left(S_t\text{-}a\right)dt+\sigma dW_t, \tag{5}$$

Where (W_t) is a standard Brownian motion, σ the volatility of the process, and λ the velocity with which the process reverts to its long term mean *a*.

In electricity markets, prices show strongly mean-reverting behavior so that estimates λ are quite large. Typical characteristic times for mean reversion are within a few days. Therefore, this model has the major drawback that futures prices are nearly constant over time, since under the assumption of (5) the futures price is given by

$$F_{t,T}=\mathrm{a}\left(1\text{-}e^{0\text{-}\lambda(T-t)}\right)+S_t e^{-\lambda(T-t)} \tag{6}$$

For this reason, several authors suggest a two-factor model, see e.g. [8], [9] and [10]. In [9] a model of the form

$$dS_t=\text{-}\lambda\left(S_t-Y_t\right)dt+\sigma dW_t \tag{7}$$

is suggested, where Y_t is a Brownian motion. A similar model is given in [10], where commodity prices are described in the form

$$S_t=\exp\left(X_t+Y_t\right), \tag{8}$$

where (X_t) is an Ornstein-Uhlenbeck process responsible for the short-term variation and (Y_t) is a Brownian motion describing the long-term dynamics. The model we will introduce in and can be considered as an extension of the ideas of [10].

All models considered so far did not consider seasonality. Some authors simply neglect this serious difficulty. Others propose to use deterministic seasonality described by sinusoidal functions, see [11], [12], [13] and [10]. In [14] it is suggested to use equation (5) with a long-run mean a_t describing the seasonal patterns. A general deterministic seasonality is proposed in [14] and [15]. Here, the spot price is modeled as

$$S_t=f\left(\mathrm{t}\right)+X_t \ or\ S_t=\exp\left(f\left(\mathrm{t}\right)+X_t\right) \tag{9}$$

with an arbitrary deterministic function $f(t)$ and a mean-reverting stochastic process X_t

In our approach, the deterministic component $f(t)$ is specified by the load forecast `t and additional stochastic behavior is introduced by the use of SARIMA models for the time series of load and prices.

There are also different attempts to account for price spikes. One possibility to cope with spikes is the introduction of jump terms, see [4], [14], and [17]. The main criticism for these models is that under the typical assumption of a jump-diffusion model a large upward jump is not necessarily followed by a large downward jump. Therefore, some authors suggest hidden Markov models, also known as Markovian regime-switching models, where it is guaranteed that upward jumps are followed by downward jumps. Such models have been considered e.g. in [18, [19], [20], [21] and [22]. Regime-switching models are very intuitive candidates for electricity price models since there are some clear physical reasons for switches of regimes such as forced outages of important power plants. On the other hand, it seems to be difficult to combine regime-switching with seasonality.

Another approach is motivated by the economic background for price spikes. Prices are determined mainly by supply and demand (load). Therefore, the non-linear relation between load and price should be taken into account in the model. This non-linear transformation is called the 'power stack function' in [23] and [24]. They suggest an exponential function for that purpose. A similar model for spot prices has recently been considered in [1], where the relation

$$S_t=f\left(X_t\right) \tag{10}$$

is suggested with X_t being an Ornstein-Uhlenbeck process, and f a power function. We also prefer an approach based on the power stack function, since there is a natural interpretation of this non-linear transform in terms of the merit order curve.

III. ELECTRICITY DERIVATIVE PRICING

A. Electricity Options

The power industry had been utilizing the idea of options through embedded terms and conditions in various supply and purchase contracts for decades, without explicitly recognizing and valuing the options until the beginning of the electricity industry restructuring in the U.K., the U.S. and the Nordic countries in the 1990s. The emergence of the electricity wholesale markets and the dissemination of option pricing and risk management techniques have created electricity options not only based on the underlying price attribute (as in the case with plain vanilla electricity call and put options), but also other attributes like volume, delivery location and timing, quality, and fuel type.

A counterpart of each financial option can be created in the domain of electricity options by replacing the underlying of a financial option with electricity [25] for introduction to various kinds of financial options). Here, we describe a sample of electricity options that are commonly utilized in risk management applications in the generation and distribution sectors. These options usually have short- to medium maturity times such as months or a couple of years. Options with maturity times longer than 3 years are usually embedded in long-term supply or purchase contracts, which are termed as structured transactions.

Now, we turn to the pricing of a European call option written on the electricity spot price. Recall, that a European option is a contract that gives the buyer the right to buy/sell the underlying commodity at some future date t (called maturity) at a certain price K (called the strike price). First, we find the pricing measure Q^{λ}. Like Merton (1976) in the context of jump-diffusion processes, we assume that the dynamics of spikes and drops are the same in the actual and pricing measures. We start with finding the spot price dynamics under λ parameterization.

Let $\lambda(u)$ be a deterministic function square-integrable on $u \in [0, T_{\max}]$, where $T_{\max}$ is a time horizon long enough to contain all maturities of derivatives quoted in the market, and introduce a new process W_t^{λ}:

$$W_t^{\lambda} = W_t + \int_0^t \frac{\lambda(u)}{\sigma_b} du, \qquad (11)$$

where σ_b is the volatility of the base regime. From the Girsanov theorem, we have that W_t^{λ} is a Wiener process under a new measure Q^{λ} defined as

$$\frac{dQ^{\lambda}}{dQ} = \exp\left[-\int_0^{T_{\max}} \frac{\lambda(u)}{\sigma_b} dW_u - \frac{1}{2}\int_o^{T_{\max}} \left(\frac{\lambda(u)}{\sigma_b}\right)^2 du \right] \qquad (12)$$

with the filtration F_t^W, being the natural filtration of the process W_t. Now, the base regime process $X_{t,b}$ can be rewritten as:

$$dX_{t,b} = [\alpha - \lambda(t) - \beta X_{t,b}]dt + \sigma_b dW_t^{\lambda} \qquad (13)$$

and the expected future spot price is given by:

$$E^{\lambda}(P_t \mid F_0) = p_{bb}^{(t)} \begin{bmatrix} X_0 e^{-\beta t} + \frac{\alpha}{\beta}\left(1 - e^{-\beta t}\right) \\ -\int_0^t e^{-\beta(t-u)} \lambda(u) du \end{bmatrix} + p_{bs}^{(t)} \left(e^{\mu_s + \frac{1}{2}\sigma_s^2} + c_s \right) p_{bd}^{(t)} \left(-e^{\mu_d + \frac{1}{2}\sigma_d^2} + c_d \right) + g_t . \qquad (14)$$

The function $\lambda(t)$ can be calibrated to the market forward prices so that $E^{\lambda}(P_t \mid F_0) = f_0^t$, e.g. by using some fitting procedure (like the least-squares minimization). Alternatively, one can find the risk premium and then use the relation between the market price of risk $\lambda(t)$ and the risk premium:

$$p_{bb}^{(t)} \int_0^t e^{-\beta(t-u)} \lambda(u) du = RP(t), \qquad (15)$$

which is a simple consequence of the fact that $RP(t) = E(P_t \mid F_0) - E^{\lambda}(P_t \mid F_o)$, formula (15) and Ito's lemma.

Now, the price of a European call option written on the electricity spot price can be derived.

Option price formula. If the electricity spot price P_t is given by the MRS model then the price of a European call option written on P_t with strike price K and maturity T is equal to:

$$C_T(K) = e^{-rT} \begin{bmatrix} p_{bb}^{(T)} C_{T,b}(K) + \\ p_{bs}^{(T)} C_{T,s}(K) + p_{bd}^{(T)} C_{T,d}(K) \end{bmatrix} \qquad (16)$$

Where $$C_{T,b}(K) = \frac{s}{\sqrt{2\pi}} \exp\left(-\frac{(K' - m)^2}{2s^2} \right) + (m - K')\left[1 - \Phi\left(\frac{K' - m}{s} \right) \right] \qquad (17)$$

$$C_{T,s}(K) = \mathrm{II}_{\{K' > C_s\}} \begin{Bmatrix} \exp\left(\mu_s + \frac{\sigma_s^2}{2} \right) \\ \left[1 - \Phi\left(\frac{\log(K' - c_s) - \mu_s - \sigma_s^2}{\sigma_s} \right) \right] \\ -(K' - C_s)\left[1 - F_{LN(\mu_s, \sigma_s^2)}(K' - c_s) \right] \end{Bmatrix} + \mathrm{II}_{\{K' \le c_s\}} \left[\exp\left(\mu_s + \frac{\sigma_s^2}{2} \right) + c_s - K' \right] \qquad (18)$$

and

$$C_{T,d}(K)=\Pi_{\{k'<c_d\}}\left\{\begin{array}{l}-\exp\left(\mu_d+\frac{\sigma_d^2}{2}\right)\Phi\\ \left[\frac{\log(c_d-K')-\mu_d-\sigma_d^2}{\sigma_d}\right]+\\ (c_d-K')F_{LN(\mu_d,\sigma_d^2)}(c_d-K')\end{array}\right\} \quad (19)$$

Further, $K'=K-g_T, m=X_0e^{-\beta T}+\frac{\alpha}{\beta}\left(1-e^{-\beta T}\right)-\int_0^T e^{-\beta(T-u)}\lambda(u)du,\ s^2=\frac{\sigma_b^2}{2\beta}\left(1-e^{-2\beta T}\right)$ and $F_{LN(\mu,\sigma^2)}$ is the cumulative distribution function of the log-normal distribution with parameters μ and σ^2.

Here, we assume that the option is settled in an infinitesimal period $[T,T+\Delta]$. However, in practice, the electricity spot price usually corresponds to delivery during some period (e.g. an hour, a day) and, hence, the maturity of the option should be specified on the same timescale. On the other hand, the analyzed spot price quotations usually represent some delivery period. For instance, if the considered data is quoted daily, then the maturity of the option would be also given in daily timescale and would correspond to daily delivery.

B. Electricity Forwards

Electricity forward contracts represent the obligation to buy or sell a fixed amount of electricity at a pre-specified contract price, known as the forward price, at certain time in the future (called maturity or expiration time). In other words, electricity forwards are custom-tailored supply contracts between a buyer and a seller, where the buyer is obligated to take power and the seller is obligated to supply. The payoff of a forward contract promising to deliver one unit of electricity at price F at a future time *T* is:

Payoff of a Forward Contract (S_T-F)

Where S_T is the electricity spot price at time *T*. Although the payoff function (1) appears to be the same as for any financial forwards, electricity forwards differ from other financial and commodity forward contracts in that the underlying electricity is a different commodity at different times. The settlement price S_T is usually calculated based on the average price of electricity over the delivery period at the maturity time *T*.

Probably, the most popular electricity derivatives are the forward contracts. Recall that a forward contract is an agreement to buy (sell) a certain amount of the underlying (here MWh of electricity) at a specified future date. The settlement of the contract can be specified in two ways: with the physical delivery of electricity or with only financial clearing. Both types of settlements are in the following called delivery. Denote the price at the time t of a forward contract with delivery at the time T by f_t^T. Since the cost of entering a forward contract is equal to zero, the expected future payoff under the pricing measure should fulfill:

$$E^{\lambda}\left(P_T-f_t^T\,|F_t\right)=0, \quad (20)$$

what implies that $$f_t^T=E^{\lambda}\left(P_T\,|\,F_t\right). \quad (21)$$

Observe, that now we define the price of a forward contract at any future date t. This is motivated by the fact that the valuation at time 0 of an option written on a forward contract requires the knowledge about the forward price dynamics at the option's maturity t.

Forward price formula. If the electricity spot price P_t is given by the MRS model, then the price at the time t of a forward contract written on P_t with delivery at the time T is given by the following formula

$$f_t^T=P\left(R_{[T]}=b\,|\,F_t\right)\left[\begin{array}{l}E^{\lambda}\left(X_{t,b}\,|\,F_t\right)e^{-\beta(T-t)}+\\ \frac{\alpha}{\beta}\left(1-e^{-\beta(T-t)}\right)-\\ \int_t^T e^{-\beta(T-u)}\lambda(u)du\end{array}\right]+$$
$$P\left(R_{[T]}=s\,|\,F_t\right)\left(e^{\mu_s+\frac{1}{2}\sigma_s^2}+c_s\right)+ \quad , \quad (22)$$
$$P\left(R_{[T]}=d\,|\,F_t\right)\left(c_d-e^{\mu_d+\frac{1}{2}\sigma_d^2}\right)+gt$$

where $P\left(R_{[T]}=i\,|\,F_t\right)=\sum_{j\in\{b,s,d\}}P\left(R_{[T]}=i\,|\,R_{[t]}=j\right)\Pi_{\{R_{[t]}=j\}}$ Note that in the above formula $E^{\lambda}\left(X_{t,b}\,|\,F_t\right)$ is used, since this expectation depends on the state process value at a time t. Namely, if $R_t=b$ then $E^{\lambda}\left(X_{t,b}\,|\,F_t\right)=X_{t,b}=X_t$. On the other hand, if at time t a spike or a drop occurred then $E^{\lambda}\left(X_{t,b}\,|\,F_t\right)=E^{\lambda}\left(X_{t,b}\,|\,F_{t-1}\right)$ and again this expectation is dependent on R_{t-1} value.

When deriving the forward price dynamics, we must remember that the properties of the obtained model should comply with the observed market prices. One of the most pronounced features of the market forward prices is the observed term structure of volatility, called the Samuelson effect. Precisely, the volatility of the forward prices is quite a law for distant delivery periods, however, it increases rapidly with approaching maturity of the contracts. Here, the forward price volatility is described by the part $P\left(R_{[T]}=b\,|\,F_t\right)E^{\lambda}\left(X_{t,b}\,|\,F_t\right)e^{-\beta(T-t)}$ of the formula (22). Hence, it is specified by the volatility of the spot price base regime scaled with $e^{-\beta(T-t)}$ and the corresponding probability of switching to the base regime. Observe that the scaling factor $e^{-\beta(T-t)}$ exhibits the Samuelson effect as it increases too 1 with t approaching maturity time T. Moreover, the forward price volatility, again due to the scaling factor, is lower than the spot price volatility. This is following the behavior of the market spot and forward prices.

Electricity forward contracts listed on energy exchanges are usually settled during a certain period (a week, a month, a year, etc.). Denote the price at the time t of a forward contract settled during the period $[T_1,T_2]$ $f_t^{[T_1,T_2]}$. The latter is the mean price of forwarding contracts with delivery during the period $[T_1,T_2]$, namely:

$$f_t^{[T_1,T_2]}=\int_{T_1}^{T_2}w(T_1,T_2,T)\,f_t^T dT=\int_{T_1}^{T_2}w(T_1,T_2,T)\,E^{\lambda}\left(P_T\,|\,F_t\right)dT \quad , \quad (23)$$

where $w(T_1,T_2,T)$ is the weight function representing the time value of money. The form W depends on the contract specification. For contracts settled at maturity, we have $w(T_1,T_2,T)=\frac{1}{T_1-T_2}$, while for instant settlement $w(T_1,T_2,T)=\frac{re^{-rT}}{e^{-rT_1}-e^{-rT_2}}$, where $r>0$ is the interest rate (Benth et *al.*, 2008a). The price $f_t^{[T_1,T_2]}$ can be obtained from formulas (22) and (23). Indeed, we have:

$$
\begin{aligned}
f_t^{[T_1,T_2]} = {} & E^{\lambda}\left(X_{t,b} \mid F_t\right)\int_{T_1}^{T_2} w(T_1,T_2,T)\, P\left(R_{[T]}=b \mid F_t\right) e^{-\beta(T-t)}dt + \\
& \int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=b \mid F_t\right)\left[\frac{\alpha}{\beta}\left(1-e^{-\beta(T-t)}\right)-\int_t^T e^{-\beta(T-u)}\lambda(u)du\right]dT + \\
& \left(e^{\mu_s+\frac{1}{2}\sigma_s^2}+c_s\right)\int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=s \mid F_t\right)dT + \\
& \left(c_d-e^{\mu_d+\frac{1}{2}\sigma_d^2}\right)\int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=d \mid F_t\right)dT + \\
& \int_{T_1}^{T_2} w(T_1,T_2,T)\,g_T dT
\end{aligned} \tag{24}
$$

C. Options Written on Electricity Forward Contracts

Finally, we find an explicit formula for a European call option written on a forward contract delivering electricity during a specified period. Observe, that the forward price $f_t^{[T_1,T_2]}$ depends on the spot price at the time t and, as a consequence, also on the state process value at the time t.

We consider an option written on an electricity forward contract with settlement during a specified period, as it is the most popular specification of electricity options on energy exchanges. For example, in the EEX market, there are options written on forwarding contracts with monthly, quarterly, and yearly settlement periods. The maturity of such options is set to the fourth business day before the beginning of the underlying contract's settlement period.

Price formula for an option written on a forward contract. The price of a European call option with strike price K and maturity t written on a forward contract with delivery during the period $[T_1,T_2]$ is equal to:

$$
\begin{aligned}
Cf_t^{[T_1,T_2]}(K) = e^{-rt}\Bigg\{ & A_0(b)C_{t,b}\left(\frac{K-B_0(b)}{A_0(b)}+g_t\right) P\left(R_{[t]}=b \mid R_0=b\right) + \\
& \sum_{i\in\{s,d\}}\sum_{k=1}^{[t]}\Bigg[A_k(i)C_{[t]-k+1,b}\left(\frac{K-B_k(i)}{A_k(i)}+g_{[t]-k+1}\right) \\
& \times P\left(R_{[t]}=i, R_{[t]-1}\neq b,\dots,R_{[t]-k}=b \mid R_0=b\right)\Bigg]\Bigg\}
\end{aligned} \tag{25}
$$

Where

$$
A_k(i)=\int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=b \mid R_{[t]}=i\right)e^{-\beta(T-[t]+k-1)}dT, \tag{26}
$$

$$
A_0(b)=\int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=b \mid R_{[t]}=b\right)e^{-\beta(T-t)}dT, \tag{27}
$$

$$
\begin{aligned}
B_k(i) = {} & \int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=b \mid R_{[t]}=i\right)\times \\
& \left[\frac{\alpha}{\beta}\left(1-e^{-\beta(T-[t]+k-1)}\right)-\int_{[t]-k+1}^{T} e^{-\beta(T-u)}\lambda(u)du\right]dT + \\
& \left(e^{\mu_s+\frac{1}{2}\sigma_s^2}+c_s\right)\int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=s \mid R_{[t]}=i\right)dT + \\
& \left(c_d-e^{\mu_d+\frac{1}{2}\sigma_d^2}\right)\int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=s \mid R_{[t]}=i\right)dT \\
& +\int_{T_1}^{T_2} w(T_1,T_2,T)\,g_T dT
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
B_0(b) = {} & \int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=b \mid R_{[t]}=b\right) \\
& \left[\frac{\alpha}{\beta}\left(1-e^{-\beta(T-t)}\right)-\int_t^T \frac{\alpha}{\beta}\left(e^{-\beta(T-u)}\lambda(u)du\right)\right]dT + \\
& \left(e^{\mu_s+\frac{1}{2}\sigma_s^2}+c_s\right)\int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=s \mid R_{[t]}=b\right)dT + \\
& \left(c_d-e^{\mu_d+\frac{1}{2}\sigma_d^2}\right)\int_{T_1}^{T_2} w(T_1,T_2,T)\,P\left(R_{[T]}=d \mid R_{[t]}=b\right)dT \\
& +\int_{T_1}^{T_2} w(T_1,T_2,T)\,g_T dT
\end{aligned} \tag{29}
$$

And $C_{t,b}(K)$ is the "base regime part" of the price of a European call option written on the electricity spot price with maturity t and strike K, see equation (29) with $T=t$.

IV. PRICE RISK MODEL

Electricity has proven to be the most volatile commodity, and it is not the exception in the Colombian competitive electricity market. It makes it compulsory to develop appropriate risk management to maximize agents' benefits and minimize the corresponding uncertainty upon them. Among the risks that firms have to handle, are a macroeconomic risk, price risk, market risk, credit risk, regulatory risk, country risk, and quantity risk. The last one is a non- tradable risk, and an implicit feature of electricity, which has to do with the amount of energy that will be demanded in the future, which follows a stochastic process as well as the spot price. This situation directly impacts the firm's revenues and makes it necessary to include it when designing the hedging portfolio.

Financial theory has developed research studies to find how to address this problem. In [26] electricity risk management is handled by a multi-market trading approach, while other references like [27] get to focus on best risk management trough forward on-peak and off-peak contracts. Nevertheless, the electricity derivatives are increasingly studied and used around the world to manage the financial risks and resource adequacy of power markets, like it is advocated in [28]. Particularly [29] proposed a financial call option to hedge against critic hydrologic scenarios in the Colombian electricity system by ensuring generation adequacy.

Further, derivative instruments have been developed to also handle the quantity risk, like swing options, weather derivatives, interruptible contracts, among other instruments named in [30]. The features of these derivative products make them be usually traded over the counter (OTC); therefore, they are low liquidity instruments, which is why they are not regarded in this paper. However, references [31] and [32] study quantity risk.

Oum, Deng, and Orean [33] deal with the static hedging problem of an LSE who has to serve an uncertain electricity demand q at a regulated fixed price r in a single period from 0 to 1. Besides, the LSE procures the electricity to serve his customers, from the wholesale market at a spot price p. Hence the profit of the LSE would be:

$$y(p,q)=(r-p)\cdot q \tag{30}$$

To protect himself against price risk, the LSE can take a long position in q^{-9} forwarding contracts at a fixed forward price F. However, the LSE will face another risk that arises from the fact that demand $\overline{q}$ may vary from the expected value at time 0 to the actual realized value at time 1. Then, if the actual demand realized by the retailer agent is $\overline{q}+\Delta q$, then, the share of the profit in (28), that is at risk is: $(r-p)\cdot\Delta q$, where Δq could be different to zero (gains/losses).

To deal with this hedging problem, the authors derive the optimal hedging portfolio as a function of the spot price p. That is:

$$Y(p,q)=(r-p)q+x(p) \tag{31}$$

where $x(p)$ is the optimal hedging portfolio as a function of the spot price p.

Regarding the LSE's preferences and risk aversion profile, it is necessary to identify its utility function U over total profit Y. In turn, because of the positive correlation between price and demand, which will be evident later in the paper for the Colombian electricity market, there exists a joint probability function $f(p,q)$ defined on the probability measure P, which characterizes the behavior of p and q at time 1. On the other side, Q is a risk-neutral probability measure by which the hedging instruments are priced and $g(p)$ is the probability density function of p under Q. Keeping this in mind, the optimization problem is formulated as follows:

$$\max_{x(p)} E\left[U\left(Y\left(p,q\right)\right)\right] \tag{32}$$

$$\text{s.t. } E^{Q}[x(p)]=0 \tag{33}$$

where $E[\cdot]$ and $E^{Q}[\cdot]$ denote expectations under probability measures P and Q, respectively. In (30) the constraint implies that the hedge portfolio $x(p)$ is self-financing, that is, the LSE can borrow funds in the money market, to purchase the derivative instruments needed to obtain the maximum expected utility over the total profit $Y(p,q)$. This constraint also means that there are no arbitrage opportunities through this hedging portfolio, under a constant risk-free rate. The optimization process yields as a result of the optimal payoff function $x^{*}(p)$.

Through an extension of the fundamental calculus theorem, it is demonstrated that any twice continuously differentiable function can be written as follows, for fixed value F:

$$x(p)=x(F)\cdot 1+x'(F)\cdot(p-F)+\int_0^F x''(K)\cdot(K-p)^{+}\,dK+\int_F^{\infty} x''(K)\cdot(p-K)^{+}\,dK \tag{34}$$

The expression $(\cdot)^{+}$ in the above equation is equivalent to the function $max(\cdot,0)$. It is important to note that in the expressions above 1, $(p-f)$, $max(K-p,0)$, $max(p-K,0)$ correspond to the payoff profile of a bond, forward contract, put option, and call option, respectively. In this sense, and remaining the LEGO© approach theory presented, with $x(F)$ units in bonds, $x'(F)$ units of forwarding contracts, $x''(K)dK$ units of put options with the strike price $K(K<F)$, and $x''(K)dK$ units of call options with the strike price $K'(K'>F)$, it is possible to replicate the resultant optimal hedging portfolio $x^{*}(p)$ from the optimization process. This financial derivative has an underlying asset the electricity spot price.

Viewed from this angle, to replicate the optimal function $x^{*}(p)$, the equation (34) implies that it is necessary to have a set of continuum strike prices for both put and call options. Since markets are incomplete, there are no markets with that amount of strike prices on board, and assuming that there is only n put options and m call options available in the market, Oum, Deng, and Orend proposed a portfolio compounded by $x(F)$ units of bonds, $x'(F)$ units of forwarding contracts, $\frac{1}{2}(x'(K_{i+1})-x'(K_{i-1}))$ units of put options with strike prices $K_i, i=1,...,n$ and $\frac{1}{2}(x'(K_{j+1})-x'(K_{j-1}))$ units of call options with strike prices $K_j, i=1,...,m$. The errors of this replicating strategy are calculated depending on the range in which spot price is realized at time 1.

V. MODEL ESTIMATION UNDER THE PHYSICAL MEASURE

The non-storability feature of electricity along with the steeply rising supply and the inelastic demand curve, both schematically represented in Figure 1, makes the electricity price p and the electricity demand q to be positively correlated. It happens in this way in the Colombian electricity market, since the dispatch is carried out in order of merit, so, when demand rises during on-peak hours, it forces the system to put in operation a more expensive generation resource, increasing the spot price as well.

Additional to the fact explained above, throughout summer seasons, the hydropower plants which are technologies with lower variable costs, reduce their power production due to natural water inflows shortages and of course to a diminishment in water reserves into reservoirs. This turns out in a higher marginal system cost, so an increment in demand translates into an increase in the spot price. This increment in the spot price reaches larger values during summer seasons than during winter seasons. Therefore, the correlation between p and q is bigger during dry periods in the Colombian wholesale electricity market.

The earlier reasons justify the usage of the price-quantity hedging strategy presented, to design suitable derivative instruments to be used by the agents of the electricity market.

It is clear that the advantage offered by the financial derivative products arises from the fact that is likely to achieve the maximum expected value of total profit $Y(p,q)$, together with the minimum deviation of this profit (which means less uncertainty over LSE's total profit), after building a hedging portfolio with those instruments.

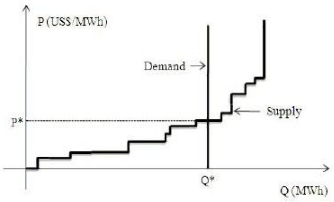

Figure 1: Supply and demand equilibria at Colombian electricity market.

To do so, a utility function to represent the risk aversion preferences of the agents like a mean-variance utility function is a good candidate to accomplish both goals at the same time. Here it is utilized the same mean-variance utility function which has been used in financial hedging literature to deal with non-tradable risk:

$$U(Y) = Y - \frac{1}{2}a\left(Y^2 - E[Y]^2\right) \quad (35)$$

where a represents the agent's risk aversion coefficient. Certainly, maximizing the expected value of the utility function in (33) $\left(E[U(Y)] = E[Y] - \frac{1}{2}aVar(Y)\right)$ is equivalent to maximize the expected value of $Y(p,q)$ and also to minimize the variance of (33), which is the advantage provided by a price-quantity hedging portfolio.

In equation (33) it is presented the optimal hedging function constrained to the mean-variance utility function. The arithmetic procedure to find this expression is developed.

$$x^*(p) = \frac{1}{a}\left(1 - \frac{g(p)/f_p(p)}{E^Q\left[g(p)/f_p(p)\right]}\right) - E\left[y(p,q)\mid p\right] + E^Q\left[E\left[y(p,q)p\right]\right]\frac{g(p)/f_p(p)}{E^Q\left[g(p)/f_p(p)\right]} \quad (36)$$

where $f_p(p)$ is the marginal density function of p under the probability measure P.

From the perspective of a clearinghouse or a market maker, who is willing to design the most adequate hedging instruments for all the participants in the market, the goodness of this model, is that equation (36) can be used to calculate the optimal payoff function for each retailer, which will determine the optimal hedging instruments needed by each particular retailer. Once this task is undertaken, and seeking to find the optimal hedging instruments for all of them jointly interacting in the market, here it is proposed to estimate a weighted average market payoff function $\bar{x}(p)$ as presented in equation (36), which will represent the joint needs of all the agents and could be used to determine general hedging instruments -suitable for all-, by no favoring big or small agents and giving to each of them the same hedging opportunities.

$$\bar{x}(p) = \sum_{i=1}^{N} w_i x_i^*(p) \quad (37)$$

Where,

$$w_i = \frac{\int_0^S x_i^*(p)dp}{\sum_{i=1}^{N}\int_0^S x_i^*(p)dp} \quad (38)$$

The variable S corresponds to a price-cap value defined under the market maker or clearinghouse criteria. Within the Colombian framework, this variable could be interpreted as the scarcity price associated with the firm energy market described above, since hedging above this price already exists given the call options related to that market.

After calculating $\bar{x}(p)$, the replicating methodology suggested is used to find the right number of bonds, forward contracts, put and call options needed to best describe the behavior of $\bar{x}(p)$.

Some of these financial instruments are currently available in the Colombian market, except the financial options. The simplest of them, bonds, can be found available in the stock market; forward contracts needed to replicate the function $x^*(p)$, could be either the current bilateral contracts (not suggested), the forward contracts that are planned to be included in MOR proposal, or even better, future contracts which are quite possibly to be launched the next year by a central chamber of counter-party risk (CRCC) to be established soon in Colombia. Since there are no financial options to replicate the optimal hedging function on the current electricity market.

Now, for the empirical analysis and the estimation procedure we use data from the Colombian electricity markets daily market representative rate value (in Colombian Pesos) the period from January 1, 1995, to December 2 2013 with 6853 observations. The fig 2 time series of the resulting price for the above Colombian electricity daily price.

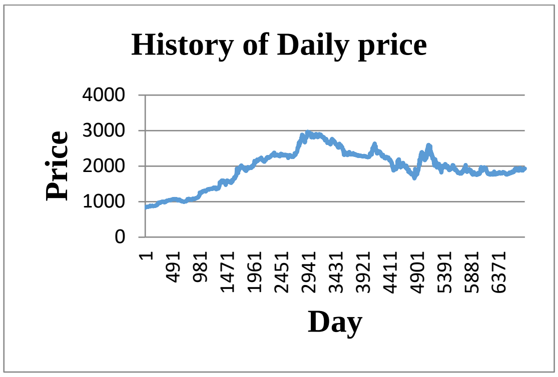

Figure 2: History of Daily price

The fig 3 gives the QQ plot for electricity daily price.

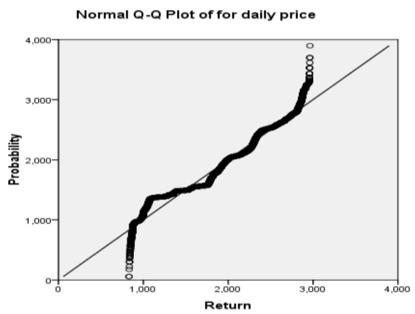

Figure 3: Normal QQ plot.

VI. CONCLUSION

From the above discussion, in this paper, we attempted to introduce an algorithm for pricing derivatives to intuition from the Colombian electricity market. Initially, we started our approach through simple stochastic models for electricity pricing. And, we derived analytical formulas for prices of electricity derivatives with different derivatives tools. 3) Then we extended short of the model for price risk in the electricity spot market. Finally, we constructed the model estimation under the physical measures for the Colombian electricity market. And this paper ends with a conclusion.

VII. REFERENCES

1. F. Black, The Pricing of Commodity Contracts, Journal of Financial Economics 3 (1976), 167 - 179.
2. D. Heath, R. Jarrow, A. Morton, Bond pricing, and the term structure of interest rates: a new methodology, Econometrica 60 (1992), 77 - 105.
3. L. Clewlow, C. Strickland, Valuing Energy Options in a One Factor Model Fitted to forwarding Prices, Working Paper, University of Sydney, April 1999.
4. G.E.P. Box, G.M. Jenkins, Time Series Analysis: Forecasting and Control, (Revised Edition) Holden-Day, San Francisco, 1976.
5. S. Koekebakker, F. Ollmar, Forward curve dynamics in the Nordic electricity market, Preprint, Norwegian School of Economics and Business Administration, 2001.
6. M. Manoliu, S. Tompaidis, Energy Futures Prices: Term Structure Models with Kalman Filter Estimation, Applied Mathematical Finance 9 (2002), 21 - 43.
7. C. L. Tseng, G. Barz, Short-term Generation Asset Valuation: a Real Options Approach, Operations Research 50 (2002), 297 - 310.
8. R. Gibson, E.S. Schwartz, Stochastic convenience yield and the pricing of oil contingent claims, Journal of Finance 45 (1990), 959 - 976.
9. D. Pilipovic, Energy Risk: Valuing and Managing Energy Derivatives, McGraw-Hill, New York, 1998.
10. E. Schwartz, J.E. Smith, Short-Term Variations, and long-term dynamics in commodity prices, Management Science 46 (2000), 893 - 911.
11. M.T. Barlow, A diffusion model for electricity prices, Mathematical Finance 12 (2002), 287-298.
12. R. J. Elliott, G.A. Sick, M. Stein, Modelling Electricity Price Risk, Preprint, University of Calgary 2003.
13. H. Geman, A. Roncoroni, A Class of Marked Point Processes for Modeling Electricity Prices, Preprint, ESSEC Business School, Cergy-Pontoise 2003.
14. S. Knittel, M. Roberts, An Empirical Examination of Deregulated Electricity Prices, Boston University, POWER Working Paper PWP-087, 2001.
15. R. Gibson, E.S. Schwartz, Stochastic convenience yield and the pricing of oil contingent claims, Journal of Finance 45 (1990), 959 - 976.
16. A. C. Harvey, Forecasting, Structural Time Series Models and Kalman Filtering, Cambridge University Press, 1989.
17. D. Heath, R. Jarrow, A. Morton, Bond pricing, and the term structure of interest rates: a new methodology, Econometrica 60 (1992), 77 - 105.
18. C. De Jong, R. Huisman, Option formulas for mean-reverting power prices with spikes, Working Paper, Erasmus University Rotterdam, 2002.
19. S. Deng, Stochastic Models of Energy Commodity Prices and Their Applications: Mean-reversion with Jumps and Spikes, Working paper, Georgia Institute of Technology, 2000.
20. R. J. Elliott, G.A. Sick, M. Stein, Modeling Electricity Price Risk, Preprint, University of Calgary 2003.
21. R. Huisman, R. Mahieu, Regime Jumps in Electricity Prices, Working Paper, Erasmus University Rotterdam, 2001.
22. V.A. Kholodnyi, The Stochastic Process for Power Prices with Spikes and Valuation of European Contingent Claims on Power, Preprint, TXU- RAG-01/00, July 2000.
23. Eydeland, H. Geman, Fundamentals of Electricity Derivatives, in Energy Modeling and the Management of Uncertainty, (pp. 35{43), Risk Books, London, 1999.
24. P. Skantze, A. Gubina, M. Ilic, Bid-based stochastic model for electricity prices: The impact of fundamental drivers on market dynamics Report, MIT Energy Laboratory (2000).
25. Hull J. Options, Futures, and other derivative, 4th Edition, Prentice-Hall, Englewood, NJ, 2000.
26. M. Liu, F. Wu, Risk management in a competitive electricity market, Electrical Power and Energy Systems Vol. 29 (2007) pp. 690 - 697.

27. R. Huisman, R. Mahieu, F. Schlichter, Electricity portfolio management: Optimal peak / off-peak allocations, Research in Management.
28. H. Chao, R. Wilson, Resource adequacy, and market power mitigation via options contracts, POWER Ninth Annual Research Conference.
29. J. Villareal, C. Soto, Ensuring generation adequacy with firm energy call options: A case study for the hydro - dependent Colombian system.
30. S. Deng, S. Oren, Electricity derivatives and risk management, Energy Vol. 31 (2006) pp. 940 - 953.
31. Y. Oum, S. Oren, S. Deng, Hedging quantity risks with standard power options in a competitive wholesale - electricity market, Special Issue on Applications of Financial Engineering in Operations, Production, Services, Logistics, and Management, Naval Research Logistics Vol. 53 (2006) pp. 697 - 712.
32. M. Prokopczuk, S. Rachev, G. Schindlmayr, S. Truck, Quantifying risk in the electricity business: A RAROC - based approach, Energy Economics Vol. 29.
33. Y. Oum, S. Oren, Optimal static hedging of volumetric risk in a competitive wholesale electricity market, Management Science (in press).